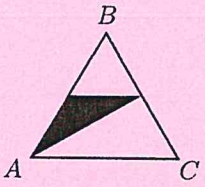
DICK SCHAFF SUPERBOWL XLV
2018 Junior High School Bomb Exam – Page 1 of 5

School _____ Team _____

Directions: **(1) Label answers with appropriate units.**
(2) Do not round or approximate answers.
(3) Write fully simplified answers on the lines provided.

1. What is the ratio of the area of the shaded triangle to the area of equilateral triangle ABC if the vertices of the shaded triangle are A and the midpoints of $\overline{AB}$ and $\overline{BC}$?



Student Name: _____ Ans: _____

2. The average of x and $2y$ is 7, and the average of x and $2z$ is 8. What is the average of x, y, and z?

Student Name: _____ Ans: _____

3. A fraction with numerator 1 is called a ***unit fraction***. Any common fraction can be written as the sum of a finite number of unit fractions; for example $\frac{9}{20}$ can be written as $\frac{1}{3}+\frac{1}{9}+\frac{1}{180}$, however it can also be written as the sum $\frac{1}{4}+\frac{1}{5}$. Using the fewest addends, write $\frac{4}{5}$ as the sum of unit fractions.

Student Name: _____ Ans: _____

4. Simplify completely: $\frac{10^8-1}{101}$

Student Name: _____ Ans: _____

5. Prime numbers of the form 2^n-1 are called *Mersenne Primes*. How many *Mersenne Primes* are there less than 500?

Student Name: _____ Ans: _____

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1. The length of a rectangle is increased by 50%. By what percent must the width be decreased so that the area is unchanged?

Student Name: _____ Ans: _____

2. What is the sum of the whole numbers from 1 to 1000?

Student Name: _____ Ans: _____

3. A number divided by one more than itself gives a quotient of $\frac{1}{11}$. What is the number?

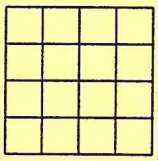
Student Name: _____ Ans: _____

4. The mathematical symbol ! is known as "factorial." When placed behind a whole number, the symbol means to multiply that whole number and all whole numbers less than it down to 1. For example, $3! = 3 \times 2 \times 1 = 6$ and $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$. What are the last two digits of the result if the following expression is simplified?

$$1! + 2! + 3! + 4! + 5! + 6! + 7! + 8! + \cdots + 98! + 99! + 100!$$

Student Name: _____ Ans: _____

5. How many squares of any size are in the image below?



Student Name: _____ Ans: _____

School _____ Team _____

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1. Express as a common fraction in reduced form: $0.\overline{12} \div 0.1\overline{2}$

Student Name: _____ Ans: _____

2. How many five-digit numbers are perfect squares?

Student Name: _____ Ans: _____

3. If the circumference of a circle is doubled in length, by what factor has the area increased?

Student Name: _____ Ans: _____

4. Calculate the absolute value of the product of the solutions to the equation $x^2 + 2017x = 2018$.

Student Name: _____ Ans: _____

5. A Magic Square is a square grid of consecutive numbers in which the sum of each row, column and diagonal is the same. This sum is known as the *constant sum*. A 3×3 Magic Square is given below as an example; it has a *constant sum* of 15. Find the *constant sum* of a 6×6 Magic Square.

8	1	6
3	5	7
4	9	2

Student Name: _____ Ans: _____

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1. $\left[\left(5^{-1}+5^{-1}\right)^{-1}\right]^{-1} =$

Student Name: _____ Ans: _____

2. Replace each letter of *ANALOG* with a digit 0 through 9 (same letters replaced by equal digits, different letters replaced by different digits) so that the resulting number is the largest such number divisible by 55. What is the resulting number?

Student Name: _____ Ans: _____

3. Solve for x: $64^{x-2} = 0.5^{3-2x}$

Student Name: _____ Ans: _____

4. Line L has equation $y = 2x + 3$. Line P has the same y-intercept as L and is also perpendicular to L. As an ordered pair, what is the x-intercept of line P?

Student Name: _____ Ans: _____

5. Four years ago, Alex was twice as old as Jake. Four years from now, Jake will be $\frac{3}{4}$ of Alex's age. How old is Alex?

Student Name: _____ Ans: _____

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1. Add any integer x to the square of $2x$ to produce an integer y. For how many values of x is y prime?

Student Name: _____ Ans: _____

2. How many distinct rectangles of any size are in the figure shown below?

Student Name: _____ Ans: _____

3. If you roll three fair six-sided dice, what is the probability that the sum of the three numbers rolled is prime?

Student Name: _____ Ans: _____

4. Suppose $f(x) = x^2 + 12$, and $m > 0$. If $f(3m) = 3f(m)$, what is the value of m?

Student Name: _____ Ans: _____

5. How many zeros are there in the square of one-trillion?

Student Name: _____ Ans: _____